relative phase velocity on exponentially inhomogeneous layers such as those listed in Table I.

Additional curves of phase constant vs thickness are available for homogeneous layers having $\mu_r\epsilon_r = 2$, 4, 8 and 15 [3]. These may be employed in solving for the phase constants of inhomogeneous layers having $\mu_r\epsilon_a = 2$, 4, 8 and 15 with the technique described herein.

Conclusion

The permittivity of a plane layer is assumed to vary continuously as a function of distance measured from the surface. Solutions for the field distributions of surface waves on the inhomogeneous layer are developed with the WKB technique. Transcendental equations for the phase velocity are derived for TE and TM modes. These equations are solved most conveniently with the aid of phase-velocity graphs which are included. The accuracy of the solution is verified by comparison with the rigorous solution for an exponential inhomogeneity.

Reasonably good accuracy is obtained even when the relative permittivity varies from 6 to 8 in a distance of 0.25 wavelength.

The formulas presented herein reduce to the rigorous solution for homogeneous layers and are accurate if the permittivity gradient is small at each point within the layer.

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UHF Backward-Wave Parametric Amplifier*

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Summary—**This paper describes a breadboard model of a UHF varactor diode backward-wave parametric amplifier that can be electronically tuned over an octave tuning range (250–500 Mc). It operates in a mode that has a relatively constant idler frequency; however, it uses two forward-wave transmission lines in contrast to the backward-wave transmission line requirement previously reported.**

A theoretical discussion on the design considerations of this mode is presented and applied to the UHF model. Measurements taken in the conventional mode of operation (output frequency equal to the input frequency) yielded voltage gain bandwidth products in excess of 100 Mc and over-all effective receiver noise temperatures of less than 140°K. Detailed measurements in the mode where the constant idler frequency is used as the output were not taken because directional filters and circulators, which are necessary in this mode, were not available.

I. Introduction

THE BACKWARD-WAVE parametric amplifier (BWPA) is a low-noise preamplifier that is capable of being electronically tuned at a rapid rate over a greater-than-octave tuning range [1]–[3]. It consists, in general, of two separate and distinct circuits that are coupled together by means of nonlinear or time-varying reactive elements.

Recently, a new class of BWPA has been evolved [4], [5] in which the center frequency of the output pass band (which is taken at the idler frequency) remains constant as the input amplification band of the amplifier is varied. This is an advantage over the conventional BWPA since it eliminates the tracking problems associated with the complex demodulator necessary to convert the normally varying output frequency to a constant IF. It thus yields an amplification system that has a greater tuning rate potential than that of the conventional BWPA. However, the realization of this amplifier mode required one of the two coupled transmission lines to have a backward-wave characteristic, which at the lower frequencies does not present any problems but presents increasing design difficulty as the frequency approaches the UHF and microwave region.

This paper proposes a new configuration that yields a nearly constant idler frequency over an octave tuning range in which both of the coupled transmission lines are forward-wave types and it presents theoretical and experimental results of a UHF model whose design was based upon this configuration.

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II. Design Consideration

A. General

To obtain parametric amplification in a backward-wave mode, it is necessary to satisfy the following three general conditions [1]:

$$\omega_p = \omega_s + \omega_i \tag{1}$$

$$\beta_p = \beta_s + \beta_i \tag{2}$$

$$(V_g)_s \times (V_g)_i < 0 \tag{3}$$

where ω, β and V_g are the angular frequency, phase propagation constant and group velocity, respectively; the subscripts p, s and i represent the pump, signal and idler waves.

To achieve a constant idler frequency it is necessary to satisfy the additional condition that

$$\left|\frac{d\omega}{d\beta}\right|_s = \left|\frac{d\omega}{d\beta}\right|_p. \tag{4}$$

The above four conditions were satisfied by using a band-pass transmission line that has a backward-wave characteristic to propagate the pump and a forward-wave low-pass transmission line to propagate the signal and idler [4], [5]. However, these conditions can also be satisfied with two coupled forward-wave transmission lines having the following properties:

1) a low-pass transmission line that propagates the signal wave
2) a band-pass transmission line that propagates the pump and idler waves
3) the ω-β characteristic of the low-pass transmission line must be shaped so that the portion of the line used to propagate the signal has a group velocity $(d\omega/d\beta)_s$ equal to that of the pump $(d\omega/d\beta)_p$.
4) In order to minimize the effect of errors in (4), it is desirable that the ω-β characteristic of the band-pass filter be shaped such that the idler frequency is located near the low frequency cutoff region where β_i is relatively small and $(d\omega/d\beta)_i$ is relatively slow.

Fig. 1 shows a pair of coupled transmission lines that have these properties and a block diagram showing the wave propagation directions.

An analysis is given below that determines the transmission line constants that are necessary to satisfy the four conditions required for high gain with a constant idler frequency. Specific lumped-constant transmission line models, similar in characteristics to microwave structures, are assumed to facilitate calculations.

B. Coupled Transmission Lines

A general equation representing the full-section ω-β characteristic of a 4-element forward-wave band-

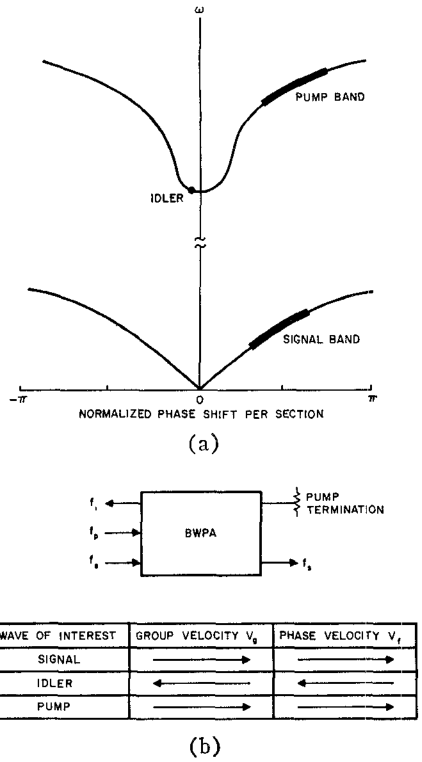


Fig. 1—ω-β and block diagram for UHF BWPA. (a) ω-β diagram. (b) Amplifier block diagram and propagation characteristics.

pass transmission line can be readily obtained [6].[1]

$$\beta = 2\sin^{-1}\left[\frac{1}{\left[1+\dfrac{x_2^2 - x^2}{m_1^2(x^2-1)}\right]^{1/2}}\right] \tag{5}$$

where

$$x_2 = \frac{f_2}{f_1}$$

$$x = \frac{f}{f_1}$$

f_1 = lower cutoff frequency
f_2 = upper cutoff frequency
m_1 = filter constant; $0 < m_1 < 1$.

By differentiating (5) with respect to x we obtain the slope

$$\frac{d\beta}{dx} = \frac{2x(x_2^2 - 1)}{m_1\left[(x^2-1) + \left(\dfrac{x_2^2 - x^2}{m_1^2}\right)\right]\left[(x_2^2 - x^2)(x^2-1)\right]^{1/2}}. \tag{6}$$

For the signal low-pass transmission line, we find [6][2]

$$\beta = 2\sin^{-1}\left(\frac{x}{x_3}\right) \tag{7}$$

[1] See pp. 174–175, series II or shunt II filter.
[2] See pp. 166–167, constant-K filter.

where

$$x_3 = \frac{f_3}{f_1}$$

f_3 = cutoff frequency of the low-pass line.

Differentiating (7),

$$\frac{d\beta}{dx} = \frac{2}{x_3\left[1 - \left(\frac{x}{x_3}\right)^2\right]^{1/2}}. \quad (8)$$

From (1)–(8) we obtain the two basic design equations

$$m_1^2 - m_1\left\{\frac{x_p x_3(x_2^2 - 1)\left[1 - \left(\frac{x_s}{x_3}\right)^2\right]^{1/2}}{(x_p^2-1)\left[(x_2^2-x_p^2)(x_p^2-1)\right]^{1/2}}\right\} + \frac{x_2^2 - x_p^2}{x_p^2 - 1} = 0 \quad (9)$$

and

$$m_1^4 - m_1^2\left\{\frac{\left[\left(\frac{x_2^2 - x_i^2}{x_2^2 - 1}\right)^{1/2} + \left(\frac{x_2^2 - x_p^2}{x_p^2 - 1}\right)^{1/2}\right]^2}{\left(\frac{x_s}{x_3}\right)^2} - \frac{x_2^2 - x_i^2}{x_i^2 - 1} - \frac{x_2^2 - x_p^2}{x_p^2 - 1}\right] + \left(\frac{x_2^2 - x_p^2}{x_p^2 - 1}\right)\left(\frac{x_2^2 - x_i^2}{x_i^2 - 1}\right) = 0 \quad (10)$$

where

$$\left.\begin{aligned} x_p &= \frac{f_p}{f_1} \\ x_s &= \frac{f_s}{f_1} \\ x_i &= \frac{f_i}{f_1} \end{aligned}\right\} \text{midband values.}$$

Simultaneous solutions of (9) and (10) have been obtained using graphical techniques. There exists a family of solutions to these equations, however, one of these, which is a good compromise between low-noise figure and circuit realizability, is

$m_1 = 0.322$ for

$x_s = 0.512, \quad x_2 = 1.55, \quad x_i = 1.12$ and $x_p = 1.44.$

Substituting these values into (5) and (7) permits the plotting of the theoretical normalized ω-β diagram shown in Fig. 2. From Fig. 2, the expected f_p and f_i that satisfy the frequency-phase gain conditions can be obtained as a function of f_s. These are plotted in Fig. 3 as a function of x_s, where it can be seen that over an octave signal-frequency range the theoretical idler-frequency variation is about ± 1.45 per cent. (This is almost identical with the variation in the backward-wave transmission line case [5].)

The minimum number of diodes required for a high gain condition can be determined from the general

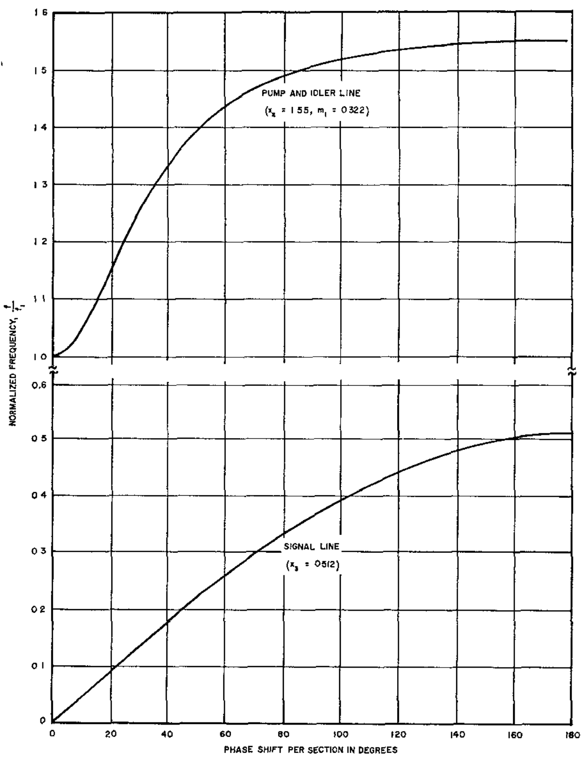


Fig. 2—Theoretical normalized ω-β diagram.

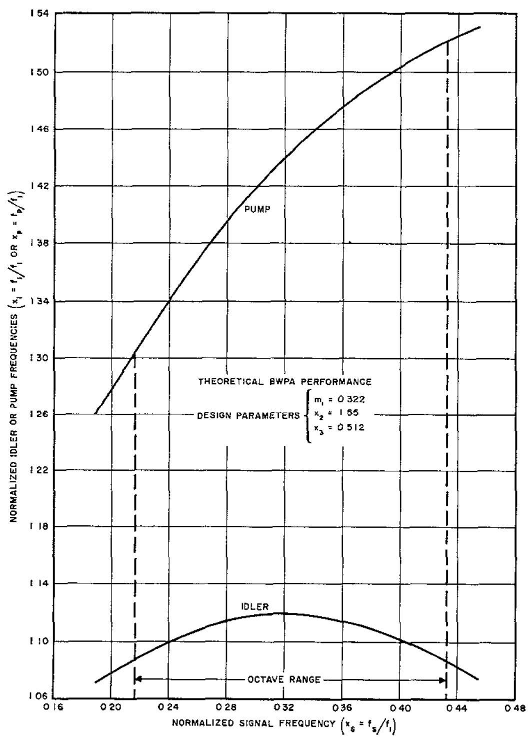


Fig. 3—Theoretical pump and idler frequencies as a function of input signal frequency.

gain equation [1].

$$G \approx \frac{1}{\left(\frac{\Delta\beta}{\xi}\right)^2 + \cos^2 \frac{\xi L}{2}} \text{ for } \left(\frac{\Delta\beta}{\xi}\right) \ll 1 \quad (11)$$

where

$\Delta\beta = |\beta_p - (\beta_s + \beta_i)|$, which is the deviation from the optimum propagation-constant condition in (2)

$\xi = \frac{C_1}{C_0}\sqrt{\beta_s \beta_i}$ = coupling constant

$\frac{C_1}{C_0}$ = diode nonlinearity;

$$C(t) = C_0 + 2C_1 \cos(\omega_p t - \beta_p L) + \cdots$$

L = length of transmission line.

To achieve high gains, $\xi L \to \pi$. Instead of using a distributed length L in (11) we can consider sections of transmission line N, where each section is an equivalent π or T network of the transmission line. Thus the number of transmission line sections is given by

$$N \geq \frac{\pi}{\left(\frac{C_1}{C_0}\right)\sqrt{\beta_s \beta_i}}$$

where N = integer.

Assuming $C_1/C_0 = 0.35$ and using the propagation constant values from Fig. 4 (an unnormalized plot of Fig. 2), it is found that about 19 diode sections are required.

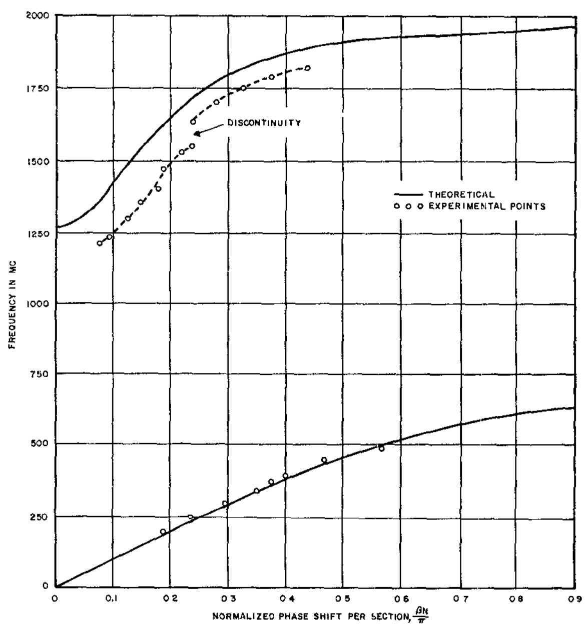


Fig. 4—Comparison of measured and calculated ω-β characteristics.

III. Experimental Data

Based upon the preceding analysis, a breadboard model of a UHF BWPA was constructed and tested. The low-pass transmission line consisted of slabline with a meander line configuration; the series inductances are made up of sections of high-impedance line and the shunt capacitances consist of parallel varactor diode pairs. The band-pass line consists of a reduced-height waveguide periodically loaded with pairs of varactor diodes in series. A photograph of the amplifier is shown in Fig. 5.

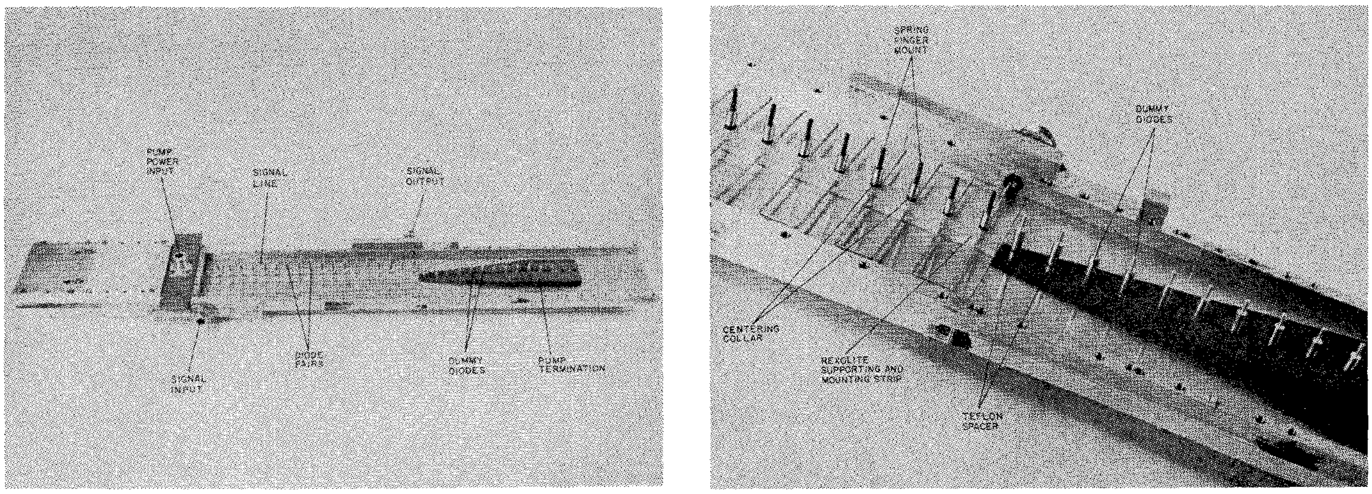
The lower and upper cutoff frequencies of the band-pass line were 1170 and 1920 Mc, respectively and the cutoff frequency of the low-pass line was 650 Mc. These cutoff frequencies yield $f_2/f_1 = 1.64$ and $f_3/f_1 = 0.543$, which are reasonably close to the design goals.

The amplifier uses 19 individually matched pairs of MA4256X varactor diodes having an average cutoff frequency of 75 kMc and an average capacitance $[C(0)]$ of 2.0 pf ± 0.1. The over-all configuration affords a high degree of isolation of the signal and pump frequencies, since the field configuration of the signal is balanced (TEM mode) and the pump field configuration is unbalanced (TE_{10} mode) [7]. In addition, the cutoff frequencies of the two lines add to the isolation. In order to use the constant idler as the output in this configuration, a circulator or directional filter must be placed at the input to the pump transmission lines in conjunction with a band-pass filter centered at the idler frequency for added isolation [Fig. 6(a) and (b)].

Amplifier tuning was accomplished by electronically varying the pump frequency and amplitude of a BWO (QK 786). No mechanical adjustments were necessary. Stable gains in excess of 20 db were obtained across a 230 to 500 Mc tuning range and the available pump power required varied from 50 to 200 mw. The measured pump frequencies required to tune the amplifier and the resulting generated idler frequencies are shown in Fig. 7. From this curve it can be seen that

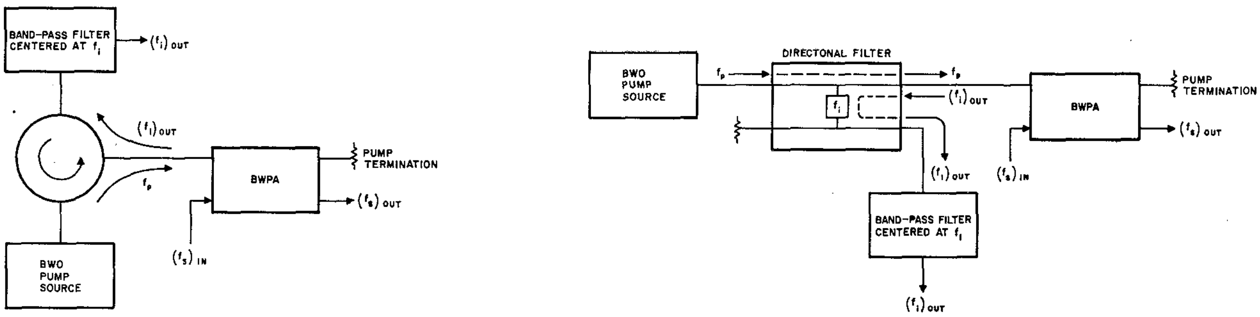
a) there is a discontinuity in the measured pump and idler frequencies. (This discontinuity is caused by a spurious resonance in the pump propagation characteristic that is exhibited in the ω-β plot of Fig. 4.)
b) the measured data deviates from the expected values. This is a result of the deviation of the "pump-idler" ω-β characteristic from its design goal (Fig. 4). The deviation is in a direction that causes a decreased idler and pump frequency, which is the condition exhibited by the measured points in Fig. 7.

From the theoretical plot of idler vs signal frequency (Fig. 7), it can be seen that the expected idler variations in the octave signal range from 250–500 Mc is ± 1.87 per cent. This is greater than the previously mentioned ± 1.45 per cent because the cutoff frequencies of the amplifier's transmission lines deviated from the design



(a) (b)

Fig. 5—BWPA with top ground plane removed. (a) Over-all view of amplifier construction. (b) Detailed view of amplifier construction.



(a) (b)

Fig. 6—Configurations for constant idler output frequency mode. (a) Circulator configuration. (b) Directional filter configuration.

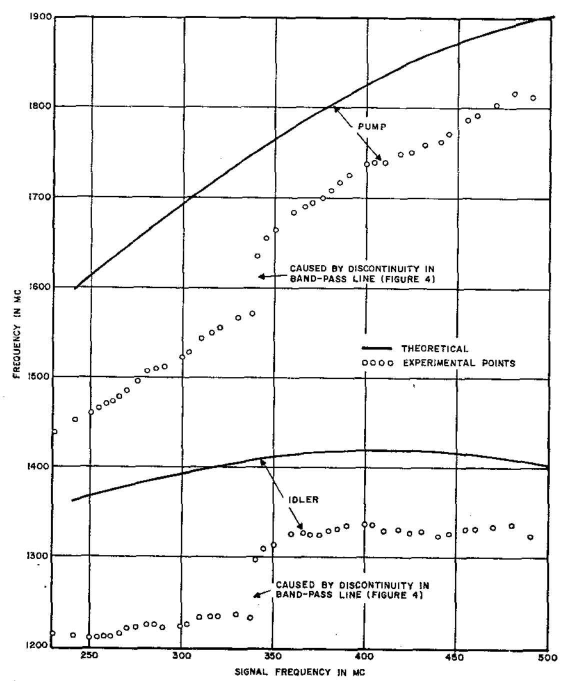


Fig. 7—Comparison of measured and calculated pump and idler frequencies as a function of input signal frequency.

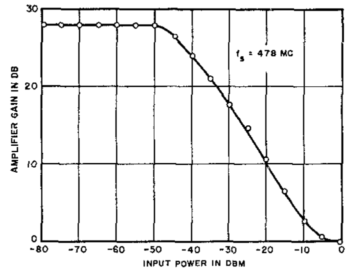


Fig. 8.

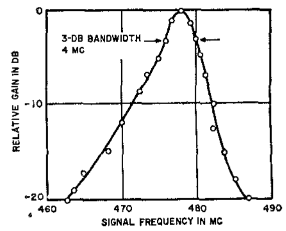


Fig. 9.

goals. Thus the 250–500-Mc frequency band is not coincident with the flattest portion of the idler curve. The measured idler variations were approximately ±5 per cent; however, this large variation is caused by a jump resulting from the ω-β discontinuity. When one considers the measured idler frequencies on either side of the discontinuity, it is clear that they exhibit the expected magnitude of variations.

A set of performance characteristics were measured on the amplifier operating in the conventional mode—output frequency equal to the input frequency. Performance data on the amplifier operating in the constant idler output were not taken because directional filters and circulators operating in this frequency range were not available. However, data taken in the conventional mode are directly applicable to the constant idler mode [5].

A gain saturation characteristic as a function of input power of the amplifier is shown in Fig. 8. The amplifier was operating at a gain of 28 db. It can be seen that the gain saturates—that is, deviates from linearity by 3 db at an input power level of about −42 dbm.

A typical bandwidth curve taken at a frequency of 478 Mc and a gain of 28 db is shown in Fig. 9. It is clear that the bandwidth is about 4 Mc; this yields voltage gain bandwidth products of greater than 100. Under the conditions of 20-db gain, the measured bandwidth was greater than 10 Mc.

Measurements were made on the effective receiver noise temperature using the Y-factor method with a hot- and cold-load noise generator (AIL Model No. 07002). Several difficulties were encountered in this measurement. One problem was caused by instabilities in the second-stage amplifier brought about by pump power leakage from the BWPA. By incorporating a double-tuned band-pass filter at the output of the BWPA in addition to an existing low-pass filter, the pump leakage was reduced sufficiently at a few frequencies to obtain valid noise-figure data. The measurements made at two frequencies are listed in Table I.

TABLE I

UHF/BWPA NOISE-TEMPERATURE DATA

Frequency (Mc)	Gain (db)	(T_e) Over-all Measured (°K)	(T_e) Second Stage Measured (°K)	$(T_e)_{BWPA}$ Calculated from Measured Data (°K)	$(T_e)_{BWPA}$ Theoretical* from f_s/f_i (°K)
365	21	128	1160	119	80
415	24	139	1210	134	87

* The minimum effective receiver noise temperature $(T_e)_{min}$ that can be obtained in a BWPA, assuming ideal diodes and no circuit losses, is [2]

$$(T_e)_{min} = \frac{f_s}{f_i} T_i$$

where T_i = temperature of the idler termination, °K.

IV. CONCLUSIONS

It has been shown that a UHF BWPA capable of operating over an octave tuning range with high-gain low-noise figures and a constant idler frequency is realizable, without the requirement of a transmission line having a backward-wave characteristic.

V. ACKNOWLEDGMENT

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